CONFIDENCE LIMITS ON NET TECTONIC ROTATION

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Abstract. Estimation of confidence limits on net tectonic rotation by an ‘overlapping circles’ method can lead to values which are too large. A simple method based upon an F-ratio test is presented which yields more accurate constraints on net rotation. An example illustrates that the limits calculated by the ‘overlapping circles’ method can be in error by more than 20%.

Net Tectonic Rotation

Paleomagnetic results from tectonically disrupted terranes often yield mean directions of magnetization which are discordant when referred to expected directions derived from a reference apparent polar wander path (APWP). One widely used method of checking for discordance employs a test for overlap or non-overlap of confidence circles about the observed and expected directions [Beck, 1980]. Discordant directions are defined as those whose rotation \( r \) or flattening \( f \) exceed the error limits \( \Delta r \) or \( \Delta f \):

\[
\begin{align*}
    r &= D_x - D_x
    \\
    \Delta r &= (\Delta D_x^2 + \Delta D_y^2)^{\frac{1}{2}}
    \\
    f &= T_y - T_x
    \\
    \Delta f &= (\Delta T_x^2 + \Delta T_y^2)^{\frac{1}{2}}
\end{align*}
\]

where

\[
\begin{align*}
    \Delta D_x &= \sin^{-1} \left( \frac{\sin \alpha_x}{\cos T_x} \right) \\
    \Delta D_y &= \sin^{-1} \left( \frac{\sin \alpha_x}{\cos T_y} \right)
\end{align*}
\]

\[
\begin{align*}
    \Delta T_x &= \alpha_x
    \\
    \Delta T_y &= \alpha_x
\end{align*}
\]

\[
\begin{align*}
    \bar{R}_o &= L_x \hat{x} + M_y \hat{y} + N_z \hat{z}
    \\
    R_o^2 &= L_x^2 + M_y^2 + N_z^2
\end{align*}
\]

\[
\begin{align*}
    L_x &= \sum_{i=1}^{n_o} \cos D_x(i) \cos I_x(i)
    \\
    M_y &= \sum_{i=1}^{n_o} \sin D_x(i) \cos I_x(i)
    \\
    N_z &= \sum_{i=1}^{n_o} \sin I_x(i)
\end{align*}
\]

\[
\begin{align*}
    \hat{x}, \hat{y}, \hat{z} \text{ are unit vectors and the } D_x(i) \text{ and } I_x(i) \text{ are the declination and inclination of the } i\text{th unit vector of the population of observed unit vectors. Obviously there}
\end{align*}
\]

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TABLE 1. Synthetic Populations

<table>
<thead>
<tr>
<th>( I_e )</th>
<th>( D_e )</th>
<th>( I_o )</th>
<th>( D_o )</th>
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<tr>
<td>45.9</td>
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<td>42.5</td>
<td>63.0</td>
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<td>1.1</td>
<td>25.2</td>
<td>16.3</td>
</tr>
<tr>
<td>50.4</td>
<td>333.5</td>
<td>24.3</td>
<td>27.8</td>
</tr>
<tr>
<td>58.4</td>
<td>30.2</td>
<td>39.9</td>
<td>39.1</td>
</tr>
<tr>
<td>29.3</td>
<td>332.7</td>
<td>52.6</td>
<td>49.3</td>
</tr>
<tr>
<td>59.2</td>
<td>19.9</td>
<td>39.0</td>
<td>51.5</td>
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<td>6.0</td>
<td>32.8</td>
<td>72.6</td>
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<td>10.4</td>
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<td>40.8</td>
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<tr>
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<td>32.1</td>
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<tr>
<td>53.2</td>
<td>27.4</td>
<td>36.7</td>
<td>64.8</td>
</tr>
</tbody>
</table>

where \( a = \cos I_e \cot I_o \cos \phi + \sin I_e \sin I_o \). For a given significance level and angle \( \phi \), we can test whether the two populations are significantly different by using the statistic:

\[
F = \frac{n - 2}{n - R_s - R_x} \left( R_s + R_x - R \right)
\]  

where \( n = n_s + n_x \). This is of course assumes that the precision parameters for each population are not significantly different [Watson, 1956]. We can define \( F_t \) as the 'threshold value' for \( F \), where the two populations become just significantly different. By substituting \( F_t \) for \( F \) in (14) and recalling that all quantities in (14) except \( a \) are invariant upon rotation, we find that the threshold is achieved when \( a \) equals

\[
\frac{1}{2R_sR_x} \left[ \left( R_s + R_x - \left( \frac{F_t(n - R_s - R_x)}{n - 2} \right) \right)^2 - R_s^2 - R_x^2 \right]
\]  

The value of \( \phi \) which corresponds to the threshold is given by

\[
\cos \phi_t = \frac{a - \sin I_e \sin I_o}{\cos I_e \cos I_o}
\]  

Since the changes in \( R \) are symmetric about the \( \bar{D}_o = 0^\circ \) value, the net tectonic rotation is given by

\[
\bar{D}_s - \bar{D}_x = \pm \phi_t
\]  

Up to now, we have considered only rotations about a vertical axis. Limits on rotation about an arbitrary axis can obviously be calculated by first rotating the chosen axis to vertical (and with it \( \bar{D}_s, I_o \) and \( \bar{D}_x, I_x \)).

Example

As an example, we consider the two populations tabulated in Table 1. From a previous section, we know that the value of \( F \) calculated from (14) must be less than or equal to 3.14 when the two populations are not significantly different. The minimum value for \( F \) will occur when \( \phi = 0 \), and from (14) we see that \( F_t = 0 \) is 0.83, so a vertical axis rotation will clearly bring the two populations into a configuration where their mean directions are not significantly different. Setting the threshold value \( F_t = 3.14 \), we see from (15) that \( a_t = 0.97 \) leading to a value of 15.4° for \( \phi_t \), the threshold value of rotation. The apparent rotation is 39.7° ± 15.4° and thus the 18.6° value calculated by the 'overlapping circles' method is 21% too large.

For a practical application, we examine the apparent rotation of the southern part of the Sierra Nevada batholith, where Kanter and McWilliams [1982] reported deflected paleomagnetic directions (\( D = 23.2^\circ \), \( I = -58.0^\circ \), \( n = 8 \), \( R = 7.928 \)) from the late Cretaceous Bear Valley Springs pluton. These data may be compared with reference directions from the Mt. Gibson pluton over 200 km to the north in the central Sierra Nevada (\( D = 329.2^\circ \), \( I = -50.3^\circ \), \( n = 19 \), \( R = 18.832 \), Frei et al., in press). The Bear Valley Springs (BVS) direction has been converted to Mt. Gibson (MG) coordinates with an axial geometric dipole transformation.

Application of the 'overlapping circles' method suggests that the Bear Valley Springs pluton has rotated 54.0° ± 12.4°
with respect to the the Mt. Gibson pluton, with an indistinguishable \((-1.3^\circ \pm 6.5^\circ\) flattening. The method presented above yields an identical \(r\) value with an 11.2° uncertainty, a reduction of about 10 percent. While the measured rotation is still statistically significant in either case, the F-ratio method provides more correct uncertainty limits, which can be critical in cases where the measured rotation is small [cf. Magill et al., 1981; Frei et al., 1984].

The confidence limits obtained from (17) are always smaller than the limits obtained from the method of Beck [1980], in agreement with the results of Demarest [1983]. An advantage of the method presented here is that exact limits can be calculated using a pocket calculator and an F-ratio table, without appeal to numerical integration.

References


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